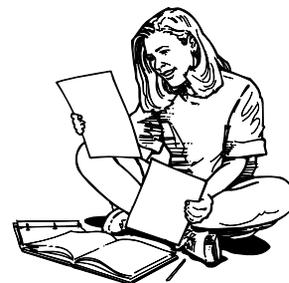


# Skip the Tricks: Build Real Understanding in Mathematics

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**T** rue confession of a high school mathematics teacher—I was something of an impostor in my high school mathematics classes. As a student, I only knew half of what was happening much of the time, but what I knew was good enough to fake it. While I could follow a process for solving problems, my mathematical understanding was one-dimensional. I could not always connect what was taught to prior knowledge, nor could I reliably extend it to real-world applications—skills that would have helped me grasp the underlying mathematics. I had high grades because my procedural skills led me to the right answers, but I lacked deep mathematical understanding. For me, and I suspect for many other educators, that “tricky” approach to learning undermined my instruction years later as a mathematics teacher.

## Developing Understanding

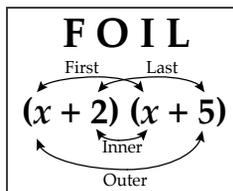
Through a lack of understanding or mathematical anxiety, students often apply the wrong tools in mathematics class. In the real world, we often develop tools to simplify

complicated tasks. Understanding the task and how the tool carries it out are critical to using the tool effectively.

My first job after high school was at an automotive repair shop in western New York where I was a general service technician. This meant that the company only trusted me to change oil and rotate tires. When I gained experience, the company asked me to take on more technical challenges. Tool vendors frequented our shop with inventory that was the auto mechanic’s pricey equivalent of a candy store. I could easily have spent a fortune on expensive specialty tools that I neither needed nor knew how to use. What I really needed were basic tools to fix common auto-related problems, tools that could be used for many different tasks. Similarly, students are often attracted to specialty tricks, when all they need is to expand their use of basic mathematical tools—and have a solid understanding of the underlying concepts—to solve common mathematics problems.

One frequently misused specialty tool is the mnemonic FOIL from beginning algebra.

This represents a method that can be used to multiply two binomial expressions, for example  $(x + 2)(x + 5)$ . When I have asked adults to recall this memory device, nearly everyone remembers that the letters stand for some variation of *First, Outer, Inner, Last*. But people rarely remember what FOIL was supposed to accomplish, which illustrates how emphasizing a particular strategy can easily overshadow its underlying concepts. This specialty tool was developed, through good intention, to give students a set of steps to get a correct answer when multiplying a binomial expression of the form  $(ax + b)(cx + d)$ .



Although FOIL applies to the product of two binomials, it cannot be applied to products of other polynomial expressions. When students latch on to a “successful” strategy without understanding its limitations, they tend either to use the strategy in the wrong problem-solving contexts—for example when adding two binomials—or fail to recognize and extend the reasoning in the strategy to similar situations, such as to find the product of two trinomials:  $(x^2 + 2x + 3)(x^2 - 2x - 3)$ .

Teaching specialty tools such as the FOIL method can prevent students from learning the key mathematical understandings behind procedures. Secondary students often fail to see that they already have the basic skills to do the problem. The FOIL strategy is a single-case extension of the standard multiplication algorithm (multiplication process) used to multiply two 2-digit numbers such as  $42 \times 45$ , or  $(40 + 2)(40 + 5)$ . The reasoning involved in multiplying any combination of two multi-digit whole numbers through the standard multiplication algorithm is a valuable and more enduring understanding that will allow students to multiply expressions with similar structures.

### The Problem with Shortcuts

Students develop some misconceptions on their own; other errors, however, stem from teacher decisions, often by-products of insufficient planning and instructional time. When the going gets tough, teachers often pull out old specialty tools to get the job done—that is,

to help students find the right answer. Emphasizing how to use tools rather than understanding how those tools work and what they do often leads to misconceptions.

I have done this myself. Once, while teaching an algebra course, I noticed that my students lacked fluency and understanding in solving linear equations with one variable. The complexity of the equations on the state exam would surely be their downfall. So I devised a process, called SOS (Simplify, One Side, Solve), that they could use to solve these equations:

- ✓ First, simplify the expressions on each side of the equal sign.
- ✓ Next, move the variables to one side of the equal sign.
- ✓ Finally, solve the resulting equation.

I was so proud of this approach that I created a poster and hung it on the classroom wall. What I did not realize was that I gave students a procedure that only applies to specific situations. I asked students to ignore the structures that might reveal more about the problem and I did not honor flexible thinking. Furthermore, SOS was designed for single-variable linear equations and it did not always maximize efficiency.

### Developing Deeper Understanding

Today, I would handle the situation in a very different way and would instead work toward developing deeper understanding. First, I would engineer a progression of single-variable linear equations that range from simple to complex and lead up to the current grade level. I would have a student start by completing a grade-level example and note his/her errors. Next, I would analyze those errors and present the student with another problem that removed some of the complexities that were challenging. I would continue presenting a simpler equation until I found a problem the student could successfully complete. Colleagues of mine have referred to this as *peeling the onion*—removing layers of complexity to find a student’s last point of success. Once teachers identify a solid foundation of understanding, they can begin building students’ knowledge by relayering complexities one at a

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time and developing their understanding of how to work with each new situation.

This kind of intervention does not happen in one class period and may take a few days or even several weeks. Though time consuming, this strategy succeeds where my SOS strategy failed and promotes conceptual understanding at each level of complexity. It honors flexible thinking through the introduction of new complexities and encourages students to consistently evaluate structure. While I believe procedure holds a place in mathematics, such procedure must be primed through the development of conceptual understanding, practice, and attention to structure and repeated reasoning.

### Overcoming Teachers' Own Misconceptions

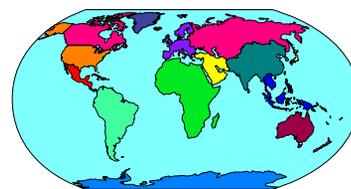
Teachers sometimes pass along their own misconceptions to students: some subtle and others significant. A misuse of language, for example, may immediately be a subtle misconception, but its lasting impact can be significant. I am rather embarrassed to admit one of my own previous misconceptions. Once,

while teaching a freshman algebra course, a student asked me what a ratio was. The idea of ratios was present in my mind, but I was not prepared to explain it. To save face, I replied that ratio is another word for fraction. I now cringe at the thought of that response! I later realized that there are significant conceptual differences between ratios and fractions. Like students, we all have our misconceptions and sometimes inadvertently transfer them to our students.

Realizing the presence of misconceptions is the first step toward overcoming them. We must communicate precisely, refine our knowledge, and reflect on how our instructional practices may contribute to the problem. It is important to remember that teachers best serve students by teaching for understanding, rather than for simply arriving at an answer. Teaching mathematics is not about overloading a student's toolbox; it is about filling the box with the necessary tools, ones that are easy to use, understandable, and applicable to a wide range of situations that span grade levels and grade bands. 📧

## Where in the World Is the ComMuniCator?

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In order to highlight CMC members and their adventures, we started a new feature in the June 2018 issue called "Where in the World Is the ComMuniCator?" We invite CMC members to take a picture of themselves in an interesting setting, while holding a copy of a recent ComMuniCator. Send a high resolution, digital photo of yourself holding a recent issue of the ComMuniCator and including a short explanation of the setting to **cmc-communicator@sbcglobal.net**.

Your photo may be printed in a future issue of the ComMuniCator. 📧

Christopher Danielson and Annie Fetter are pictured at Math On-A-Stick, a large-scale family math event that runs all 12 days of the Minnesota State Fair. Also in attendance (but not pictured) were CMC members Peg Cagle, Shelley Carranza, Casey McCormick, and Denis Lantsman.

